Preschool and Early Math Instruction: A Developmental Approach

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Imagine yourself observing a traditional classroom during math period. What do you suppose would be going on? You might hear the entire class chanting responses to the teacher's rapid-fire presentation of addition or subtraction facts. Or you might see all the children working on the same page of a workbook containing row after row of numerical combinations. The teacher very likely would have readily available ditto sheets of problems similar to those found in the workbook assignment for children unfortunate enough to have completed their assignment before the end of the period.

Now, imagine yourself in a classroom where modern math is being taught. What do you suppose would be a typical lesson? The following example might serve as a prototype.

> [The teacher asks:] "Why is 2 + 3 =3 + 2?"
>
> Unhesitatingly the students reply, "Because both equal 5."
>
> "No," reproves the teacher, "the correct answer is because the commutative law of addition holds."
>
> Her next question is, "Why is 9 + 2 = 11?"
>
> Again the students respond at once: "9 and 1 are 10 and 1 more is 11."

"Wrong," the teacher exclaims. "The correct answer is that by the definition of 2, 9 + (1 + 1) = (9 + 1) +1. Now, 9 + 1 is 10 by the definition of 10 and 10 + 1 is 11 by the definition of 11." (Kline 1973, p.1)

Shortcomings of Traditional and Modern Math Approaches

Of course, neither of these examples represents the best of either the traditional or modern math programs in some of our schools; our outstanding technological achievement is partial evidence that whatever methods have been used in math instruction have at least benefited some individuals.

What are the problems inherent in the traditional approach to teaching mathematics? Probably the most fundamental weakness is the tendency of teachers to force the student to solve problems by relying on the rote memorization of algorithms to given problems rather than on the attainment of a fundamental understanding of the problem and how to solve it (Kline 1973, p. 4). Most first grade texts require the child not only to use mathematical symbolic representations apart from their concrete referents, but also to arrange those abstractions in elaborate algebraic formulas, and then render the answers. Analyses by developmental psychologists of the cognitive competence necessary to understand such sets of abstractions indicate that this kind of assignment is beyond the grasp of most first graders (Lovell 1971, p. 14). And yet, in the face of their students' repeated failures, teachers continue to expect this of them. Why? Because most teachers see no alternative to using the textbook as a guide; they continue to teach all children the same lesson at the same time irrespective of individual differences.

Another flaw in traditional math instruction is the teacher's failure to relate math content to the experiences and interests of the child, both past and present. Experiences assimilable by young children are largely concrete; consequently, when information of a semiabstract or abstract nature is presented to them, they usually do not see how it relates to that base of experience (Ausubel 1970, p. 47). Therefore, abstractions have little or no meaning to children. The meaning that a child draws from experience is also influenced by the extent to which continuity is represented in his or her math experiences. For learning free of failure to take place, prerequisites for a given learning experience must be mastered first. When math materials and lessons are not sequenced properly (simple to complex; concrete to abstract) or are unrelated to the child's interests, then failure is virtually guaranteed (Kline 1973, p. 6).

Because traditional math materials deal primarily with numbers and doing exercises, they do not "connect" with the child's background or present interests. Just one brief glance at typical traditional math materials, particularly the books, reveals that they are insufferably dull (Kline 1973, p. 13). They contain page after page of arithmetic examples with a few problems interspersed, most of which are foreign to the child's real life situation.

The most fundamental inadequacy of traditional math, however, is that it does not even take into account, let alone facilitate, the emergence of the cognitive structures a child must develop in order to grasp the meaning that underlies number relations.

A little over ten years ago, parents and educators alike were led to believe that, with the advent of modern math, a virtual renaissance in math instruction was in the making. Workshops were springing up everywhere, with universities vying with each other to be the first to offer a full-blown course in modern math and its instruction. While the movement did serve to highlight the deplorable condition of math instruction in this country, modern math hardly represents a substantial improvement over the traditional approach.

What are some of the fundamental shortcomings of modern math? First of all, as with traditional math, modern math programs require logic and reasoning far beyond the child's cognitive capabilities. There is a heavy reliance on the use of definitions and axioms to make mathematical deductions (Kline 1973, pp. 24-50).

Set theory requires a sophisticated level of classificatory behavior, including such concepts as the union of sets (5 books + 4 lemons =9 objects); the intersection of sets (5 books, one of which is red, and 3 red objects, one of which is a book—the red book being the intersection of those two sets); subsets (red chairs as a subset of chairs); empty sets (past U.S. presidents who were women); and infinite and natural numbers. The premature introduction of set theory represents yet another dimension of the disparity between the expectations of modern math instructors and the cognitive capability of the students (Kline 1973, p. 92).

A second and fatal mistake is the use of language too abstract for the child and too far removed from the child's vocabulary (Kline 1973, pp. 60-73). Appropriate use of such terms as commutative, associative, and distributive to describe properties of various mathematical operations is expected of youngsters too immature linguistically to understand their meaning.

Related to the problem of language is the assumption of modern math educators that the concrete referents of the mathematical abstractions being presented would be automatically seen and understood by the child (Kline 1973, p. 20). This, however, is not the case, and successful early education professionals are realizing that children need to have experience with concrete objects before they can manipulate symbols, and that they need to return to the concrete experience to ensure that they, indeed, make the connection.

The framers of the modern math approach have exacerbated their inadequacies by ignoring the role of pedagogy in the educative process. (Kline 1973, p. 131). An example of this is represented in their assumption that a mere parsimonious, succinct, symbolic presentation of information is all that is necessary to get children to understand math (Kline 1973, p. 132). This error is in part due to the lack of experience the framers of modern math as pure mathematicians have had with children and with math teachers. Implicit in their scheme is the assumption that children learn math just as adults do. In other words, they assume that the best approach to math instruction is precisely the approach they use as math theoreticians, even though they themselves did not learn math in that way.

Morris Kline (1973) questions the competence of math theoreticians to serve in a leadership role in revamping math instruction because they have misguided the field of math itself for quite some time. He claims that they have abandoned science, the discipline which gave rise to the need for the development of mathematics, and are now pursuing more selective and obscure abstractions unrelated to concrete reality (Kline 1973, p. 126). Modern math theorists tend to know very little about the fields of physics and engineering, while in the past the purpose of math was to serve these areas. This concern for the isolation of math instruction from experience has led Alfred N. Whitehead to state: "There can be nothing more destructive of true education than to spend long hours in the acquirement of ideas and methods which lead nowhere . . . "(Whitehead 1964, p. 189).

In summary, many of the shortcomings of the traditional approach are also found in the modern math program. Both fail to recognize that, at a very early age, children learn quantitative relationships basically through an inductive approach. It is simply beyond children at that developmental level to understand theoretical propositions, definitions, or axioms, use them as premises, and make logical deductions from them. Rather, their approach to understanding is to extract (induce) concepts from concrete experience and then ultimately apply them to similar situations. Only at a later developmental stage when they are capable of formal logic can they use a deductive inferential mode of thought.

Secondly, the language that is used must be comprehensible to the child. A major fault of instruction is the unwarranted assumption by teachers that children understand the meanings of the words they use. The greatest weakness, however, in both the traditional and the modern math programs is the failure to match the math learning experience with each child's developmental level. To do this requires the application of a theory of development and theories of teaching and curriculum consistent with it. This is precisely the challenge the Anisa Model addresses.

The Anisa Model

For the past fourteen years, the Anisa staff has been wrestling with the task of conceptualizing a developmentally-based educational system that not only provides the remedy for the ills just mentioned in math instruction, but those related to the total education of the child as well. Traditional education is characterized by activities in which children are required to memorize facts, mostly for the purpose of retrieving them for an exam. The rapid increase in knowledge and technology has made this approach obsolete. Microphotography, for instance, has been developed to the point where the entire Encyclopaedia Brittanica can be represented on a piece of microfilm the size of a 3 x 5 card. Factual information is doubling every eight years. Our educational systems are literally collapsing because of their inability to handle the information explosion and the rapid social change which accompanies it. This is one of many reasons why Anisa initiated a massive research effort fourteen years ago to create an alternative system of education that does far more than merely disseminate facts. This effort included an intensive investigation into both Eastern and Western philosophical thought, from philosophers predating Plato to those of the twentieth century.

Why was such an investigation necessary? The educational profession has recently experienced a proliferation of "innovative" schemes and ideas, mostly piecemeal efforts designed in ignorance of the nature of those for whom the innovations were intended. To avoid these pitfalls, a comprehensive approach to education was needed, one that would necessarily involve gathering together and organizing into a coherent scheme all of the pertinent data about human growth and development. However, at present, that body of information is fragmented, incomplete, and, in many instances, contradictory and somewhat confusing. Other attempts to integrate it have failed. We believe that this failure is largely due to the absence of a wider conceptual framework. Since the purpose of philosophy is to disclose the nature of something, a philosophical base of the Model was developed to provide a view of the nature of human beings and serve as a broad conceptual framework in which all the information about human growth and development might be organized and systematized into one, coherent, comprehensive scheme.

While it is beyond the purpose of this paper to fully explicate in detail the philosophical and theoretical underpinnings of the Anisa Model, it is important to present at least the broad strokes of that conceptual framework from which the Anisa approach to math instruction has been derived.

The philosophical foundation of the Model has been heavily influenced by the writings of Alfred North Whitehead, whose cosmology represents a unique synthesis of both Eastern and Western thought. This philosophical base identifies the fundamental propositions or assumptions about the nature of humanity which serve as the first principles of education. Because they are consistent with the fundamental principles of evolution, they enable us to determine our own future when properly understood and applied. Scientists have long known that the best way to make use of a fundamental law is to obey it (apply it). By applying the laws of aerodynamics, for example, we are not only able to travel from continent to continent, but interplanetary travel is also within our grasp. The same thing holds true for basic principles related to human nature.

Therefore, one of the fundamental principles of the Anisa Model identifies human potentialities as limitless. So long as the biological integrity of the human organism is functioning properly, there is no time during which the individual cannot learn something new. Each new bit of learning then increases one's capabilities for learning even more. Potential, therefore, can be created and in this sense is limitless. This is not to deny that one's genetic endowment is determined at conception. But the limit of that endowment's expression has never been conceptualized in science because of the human ability to create further potential.

A second fundamental proposition related to human nature is that reality inheres in the process of one's becoming. Whitehead indicates that the fundamental characteristic underlying the entire universe is change. Change can, however, be in two directions: toward increased entropy—which refers to change in the direction of randomness, a winding down; and toward increased order (negative entropy)—which is change away from randomness involving the reorganization of differentiated elements into higher levels of complexity, a process of evolution—a building up rather than a winding down. In humans, the process of creating more complex structures is essentially what is meant by the term "development." It is fundamental to survival because it enables the growing individual to deal more and more effectively with the environment over time, thereby becoming progressively more adaptive. Once you acknowledge that creation is characterized by change or process and not by static actuality, you presuppose potentiality; all of creation is changing from what it now is to what it potentially can be. The translation of potentiality into actuality, as defined by the philosophical basis of the Model, characterizes all growth and development in humans, and constitutes the essential dynamics of creativity.

A proposition of the Anisa theory of development identifies interaction with the environment as the means by which the process of translating potentiality into actuality is sustained. Therefore, the quality of the environment and the child's interaction with it are critical to the quality and speed of the child's growth and development. The philosophical affirmation that human potentialities are limitless poses a problem when it comes to identifying them for practical purposes. This difficulty was resolved by setting up categories by which all potentialities might be organized. The initial grouping identified two broad categories of potentiality—biological and psychological. The theory of development establishes *nutrition* as the key factor in the release of the biological potentialities and *learning* as the key factor in the release of the psychological potentialities. A child's learning is impaired if he is not healthy and if he suffers from malnutrition (Raman 1975).

Psychological potentialities have been categorized further into five areas: (1) psychomotor—which refers to the organization, movement, and control of the voluntary muscles; (2) perception—the organization of sensory stimuli, both internal and external, into recognizable patterns for interpretation; (3) cognition—which refers to thinking; (4) affective—the organization of feelings and emotions; and, (5) volition—the ability to formulate intentions and follow them through to satisfactory completion. Within each of these categories the main processes germane to the attainment of learning competence have been identified, defined, elaborately described, and theoretically justified. Developmental data pertinent to these processes have been compiled and a list of educational objectives derived from them has been articulated. Prototypical experiences which facilitate their development have been designed and tested.

A systematic investigation of all the extant theories of learning led to the identification of the basic elements characteristic of all known forms of learning, namely, *differentiation, integration,* and *generalization.* Learning, broadly defined, is the ability *to differentiate* elements of experience (psychomotor, perceptual, cognitive, affective, and volitional experiences) and *to integrate* those elements at a higher level of complexity and ultimately *to generalize* the integrations to other similar situations. Learning competence is the *conscious* ability to differentiate, integrate, and generalize. Becoming a competent learner is the ever-present objective of the child in an Anisa school. When children become competent learners, they are then in charge of their own destinies.

The processes which have been identified in each of the categories of potentiality are too numerous to mention here, but suffice it to say that they are ordered expressions of the energy of the individual in particular ways; they are not random expressions. For example, one process in the cognitive domain is classification, an ordered expression of the thinking potentialities of the organism to categorize or group items on the basis of a shared attribute or a set of shared attributes. Since attempting to deal with every stimulus on an ad hoc basis is beyond the capabilities of any human being, the ability to classify is imperative. It enables the individual to simplify his environment and therefore make sense out of it. We have thus far identified most of the pertinent processes in each of the five categories of potentiality and have developed a curriculum to strengthen each of them.

If development is defined broadly as the translation of potentiality into actuality, and interaction with the environment is the means by which that process is sustained, then teaching must be defined as *arranging environments* and *guiding the child's interaction* with those environments to achieve the objectives of the curriculum. Thus, an appraisal of any teaching act can be based on the extent to which the arrangement of the environment and the guidance of the child's interaction with it are appropriate to the objective in mind, and the degree to which the teacher facilitates the child's differentiating the elements comprising the experience, integrating those elements at a higher level, and then generalizing the integration to other situations.

The Anisa theory of curriculum is also a logical derivative of the theory of development and is defined as two sets of objectives, *content objectives* (which concern information about the world in which the child lives), and *process objectives* (which concern the development of learning competence in each of the categories of potentiality), *and what children do,* usually with the assistance of another person, *to achieve those objectives.* As content is assimilated and processes are mastered, the potentialities of the child are actualized.

There are three basic environments with which the child necessarily must interact: (1) the physical environment, which we define as everything represented in the ontological levels below humanity—animal, vegetable, and mineral; (2) the human environment—comprising all humans; and (3) the environment of unknowns—all of the unknowns one is aware of by virtue of consciousness, such as one's mortality; the future, what one is capable of becoming (unactualized potentialities); and most fundamentally, the ultimate mysteries in the universe.

When the human organism interacts with each of these environments, he responds as a total human being comprised of a biological self, capable of movement, perceptions, thoughts, feelings, and certain intentions. As he interacts, these learning capabilities function in a collaborative way, creating structures (patterned uses of energy) which comprise that particular environment. We have defined these structured patterns as *attitudes* and *values*. It is the role of education to guide the formation of attitudes and values in a way that brings the individual closer to the realities of those three environments and prevents his becoming out of touch with their essential nature.

The human capacity for abstraction has led to the utilization of basic symbol systems in structuring our relation to these three environments. For the material environment, math serves as a primary symbol system; for the human environment, language is the main symbol system for the interactional process; and for the environment of unknowns, art serves as the symbol system for attempting to present "visions" of possibilities—the unknown things we might become. The environments represent in themselves a hierarchy consistent with the various ontological levels specified by Whitehead (mineral, vegetable, animal, human, unknowns), and each higher level includes the properties of the lower (i.e., the human environment includes the properties of the material environment and the unknown includes those of both the human and material environments).

One particular property of any element of any of the three environments (whether mineral, vegetable, animal, etc.), is the capacity for internal factors of the element to determine to some extent its own behavior. In other words, one is unable to predict the behavior of elements of any of the environments entirely on the basis of external factors—a quality physicists came to understand about the material world when they rejected the mechanistic view of the universe in favor of an organismic view, a quality in human beings behaviorists in psychology still do not acknowledge. The principle which describes the internal determinants governing the activity of the components of each of the environments has been referred to as the subjectivist principle or principle of indeterminacy. It indicates that the amount of "indeterminacy" is increased with each higher ontological level. In other words, plants have more "self-determination" than rocks; animals have more than plants; and humans have more than rocks, plants, or animals. The symbol systems used by humans to mediate experience with the three environments—math, language, and the arts—accommodate the increased indeterminacy of each higher ontological level.

Each individual, then, has some say in how he will behave—how he will use his energies. As each individual interacts with the various environments over time, he uses the symbol systems to help establish relatively enduring patterns of energy use. In the Anisa Model, these patterns of energy use define the individual's values. To the degree that these values put him in touch with the essential realities of the physical, the human, and unknown environments, he will attain technological competence, moral competence, and aesthetic or spiritual competence, respectively. Spiritual competence is defined in psychological rather than religious or denominational terms and refers to the human capacity to approach unknowns or forming hypotheses or ideals which enable him to deal with his own mortality, his future, and the ultimate mysteries of the cosmos.

Developmental Prerequisites to Understanding Number

Math plays a critical role in the actualization of human potentialities because it enables one to come to grips with quantity—an ever-present part of the reality of any existing thing. Technological competence is impossible without the ability to understand the quantitative realities of the physical environment through the symbol system of math. Since math is essentially a cognitive activity, there are several basic cognitive processes which constitute the developmental prerequisites to understanding number relations—prerequisites to grasping the meaning of mathematical thought.

According to Piaget and many of his followers, there are four processes in the cognitive domain that have been determined to be particularly pertinent to understanding number relations. They are: *classification, seriation, transitivity,* and *conservation.*

Classification refers to the ability to group objects, actions, feelings, events, or ideas on the basis of their recognized similarities. To classify, one has to decide which items belong in a particular group or class by virtue of its possessing a property or properties which are the critieria which define the class. Classification makes sense out of experience because it reduces the complexities of the environment by ordering it into categories (Bruner, Goodnow, and Austin 1956). Can you imagine dealing with everything on an ad hoc basis? Suppose you had to confront every set of stimuli which we call "chair" on an individual basis, identifying and labeling each chair encountered individually rather than as a collection. It would be impossible to communicate to anyone else a particular "chair" experience. Only when one can abstract the properties which constitute "chairness" and represent them with a label symbolizing the same meaning can we effectively make sense out of "chair" experiences and communicate those experiences to others.

Developmentally, there are three basic types of classification: (1) *perceptual*—which is sorting items based on physical attributes such as color, shape, size, pitch, and texture (classification that is largely the result of perception tends to remain unchanged by subsequent experience); (2) *conceptual*—which refers to grouping items on the basis of function (this is largely determined by social knowledge from the culture such as things which are eaten, things you can ride on, or things that are worn); and, (3) *abstract conceptual*—which involves organizing events or ideas on the basis of a scheme which necessarily requires logical thought. It is this latter form of classification that tends to be modified, formed, and re-formed on the basis of logical processes.

There are different stages of classification. The first stage which might be considered a preclassificatory level is called *heaping.* This is when the child indiscriminately groups items on piles. The next stage is called *graphic collections*—organizing items on the basis of shifting criteria such as starting with the grouping of items using red as a criterion. The first item is red; the second is red and square. The criterion then shifts from red to squareness. The third item is a square that is yellow. The next item may be the same size as the yellow items. The graphic collections stage is followed by *simple sorting*—grouping objects according to a single property that is perceptually apparent such as color, shape, pitch, size, and texture. The fourth stage of classification is called *true classification;* in this stage the child is able to group objects by abstracting a property common to them all and to include all objects which fit into that category. The stage of *multiple classification* which follows involves classifying on the basis of more than one attribute or property and recognizing that an object can belong to several classes at the same time. For example, the criteria for classification may be the attributes of redness and squareness. The highest form of classification is called *class inclusion.* It involves forming subclasses of objects and including the subclasses in a larger class. For example, the contents of a bag of red and yellow wooden beads can be seen to have three classes: the two subclasses of red beads and yellow beads, and the inclusive class, wooden beads, to which they all belong.

Classification is not only important because it reduces the complexity of the world so that it can be dealt with in a manageable way, it is also important, according to Bruner, because almost all other cognitive activity presupposes the ability to group events according to their class.

Seriation refers to the ability to arrange items on the basis of their recognized differences. *Simple seriation* is ordering items on the basis of one dimension such as height, width, intensity of color, or pitch. *Multiple seriation* is simultaneously ordering items in a series on the basis of the predetermined dimensions, such as height, and width where the tallest and fattest would be the first in the series and the shortest and thinnest would be the last.

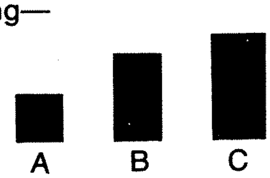
From a developmental point of view, seriation starts in a very global way by perceptually differentiating qualitative attributes of items such as color or size and identifying polarities such as big, small; loud, soft; long, short; fat, thin; etc. *Perceptual seriation* refers to the ability to order things on the basis of obvious, perceptually recognized differences. *Intuitive, progressive seriation and correspondence* (pairing items) refer to the ability to construct a series with a certain amount of trial and error effort without the intervention of another person. Concrete operational seriation refers to the ability to order objects without hesitation or the use of trial and error methods. *Ordinal correspondence* refers to the ability to integrate ordinal and cardinal relationships between the elements in two series of items. A child who has mastered this understands that in the series 1, 2, 3, 4, 5, the "3" represents both the third position (ordinality) and has the numerical value 3 (cardinality). Here we see the relationship between classification and seriation with respect to understanding number: the abstract quantitative attribute of a group is the class (its cardinal aspect), and the order of that class with respect to other items in the series, based on the recognized differences between them, is the result of seriation (the ordinal aspect of number).

Furthermore, seriation and classification tasks often entail understanding some kind of *correspondence* between two sets of items. Correspondence involves pairing objects on the basis of some common relationship among the pairs. The nature of this relationship is dependent upon whether correspondence is combined with a symmetrical or asymmetrical order. If the relationship is symmetrical (e.g., four identical apples paired with four identical dishes), the child performing the task could pair any apple with any dish, discovering in the process four pairs of elements. Because the pairs are equivalent (each apple-and-dish pair is like the others), the order involving classification is irrelevant. This is cardinal correspondence since the pairing is established irrespective of order.

When correspondence is combined with an asymmetrical order of graded differences (seriation), the nature of the pairing is determined by the position of each element in the series (e.g., five flowers of graded sizes paired with five vases of graded sizes). As Piaget (1965) states:

> . . . number is organized, stage after stage, in close connection with the gradual elaboration of systems of inclusions (hierarchy of logical classes) and systems of asymmetrical relations (qualitative seriations), the sequence of numbers thus resulting from an operational synthesis of classification and seriation. (p. viii)

Transitivity, implicit in seriation, is the process of inferring the quantitative relationship between two numbers from knowledge of the relationship they have with a third number. The simplest form is *transitivity of equivalence.* If John is as tall as Joan, and John is as tall as Jerry, then Joan and Jerry are the same height (A = B; A = C; therefore, B = C). A more complex form is *transitivity on the basis of differences.* If Mary is heavier than George, and George is heavier than Mark, then Mary must be heavier than Mark. Take the following illustration of serial ordering—



Since C is larger than B, one knows that C is larger than A through knowledge of B's relationship with A.

It is not known whether transitivity or seriation comes first in the development of the child, or which one strengthens the other. However, there does seem to be a sequence in the development of transitivity, with transitivity of equivalence preceding differences, and with the transitive inference based on the concept "greater than" coming before the transitive inference based on the concept "less than." According to Piaget, transitivity is a universal process which is implicated in practically all other forms of inference. Transitive relationships involving color and form are easier to discern than those involving height, weight, and number. Because children approximately nine years of age are capable of reversibility of thought, they should be able to handle transitive relationships on all levels using verbal cues rather than having items or numbers in question visually present.

Conservation is the process of reasoning that enables one to understand that certain attributes (qualitative and quantitative) remain the same or invariant while certain other attributes undergo transformation. In order for a person to understand conservation, one must recognize and establish equality between the data being considered (weight, volume, area, or number) before and after the transformation has taken place. Conservation presupposes being able to reverse a mental act in the following ways: (1) *inversion or negation,* which refers to cancelling an operation by combining the original operation with its opposite; (2) *reciprocity,* which refers to equalizing an operation by introducing an off-setting factor to compensate for the original change. There are several types of conservation: (a) mass or substance; (b) weight (same as above, but a scale is introduced); (c) length; (d) number; (e) continuous quantity; (f) discontinuous quantity; (g) order; and (h) area.

Why is conservation necessary? According to Piaget, conservation is basic to reasoning because reasoning requires permanence or consistency of definitions. Words presented here have to mean the same thing when presented again elsewhere. In the area of quantitative relationships, fiveness must equal fiveness, irrespective of where it is found and apart from its concrete referrents; otherwise the prospects of even beginning to understand the meaning of quantity is impossible. In other words, number is only intelligible if it remains identical to itself wherever it appears in whatever form. The same is true with a set as well as with measurement. Conservation, therefore, is a fundamental process underlying rational thought and hence is at the root of scientific inquiry.

In terms of a developmental order, during the period from birth to two years of age, the prerequisite to conservation, *object permanence,* is established. Object permanence refers to knowing that the existence of something remains, irrespective of whether it is perceivable (i.e., the baby comes to realize that "mother" does not cease to exist just because she leaves the bedroom where the baby is).

Object permanence is followed by perceptual *constancy*—knowing that the identity of something remains the same irrespective of the particular perspective from which it is being perceived. As an example, a saucer, which is round, may look cigar-shaped when viewed at an angle. However, we know that from the top view its roundness remains the same even though it looks cigar-shaped when we look at it from a different perspective. During the preoperational period, ages three to seven years approximately, the child comes to first understand qualitative consistencies, followed by the understanding of quantitative invariance concerning number, length, mass or substance, area, and continuous quantity.

Math educators have long recognized that grasping the basic meaning of number should precede representation using symbols. Otherwise, the child will be either confused or will memorize answers by rote rather than attain a fundamental understanding. Grasping the meaning of number relations depends on the child's ability to understand the quantitative attribute of a set of items, which involves classification; to understand the asymmetrical relationship between those items based on their recognized differences, which involves seriation; to make quantitative inferences on the basis of equivalences and differences, which involves transitivity; and to understand that the identity of the quantitative attributes as they are classified remains constant, which involves conservation. The child's ability to perform logical operations or processes is evidence of the existence of those psychological structures for ascertaining the meaning underlying quantitative relationships.

There may be other cognitive processes not yet identified which are basic to a child's understanding of number relations. Much more theoretical work is required coupled with longitudinal empirical research to determine if this is so. Because of the amount of research done to date, it is unlikely that the processes mentioned in this paper will take on less significance. Rather, I would expect that further research will make more formidable the argument justifying their inclusion in the curriculum, and continued and detailed study of each will yield for educators more useful information about their characteristics and how their development can be fostered in children.

We've come a long way in both identifying and redressing the weaknesses in both traditional and modern math programs. However, we have not arrived, nor shall we arrive easily, at the point of having the ideal math program given human proclivity to create potential, thus continuing to evolve. The task which remains is to find out in great detail how a child comes to understand number relationships. Our method is theory construction based on what we know about how people learn, the application of theory to curriculum development and teaching, and empirical verification of the theory.

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